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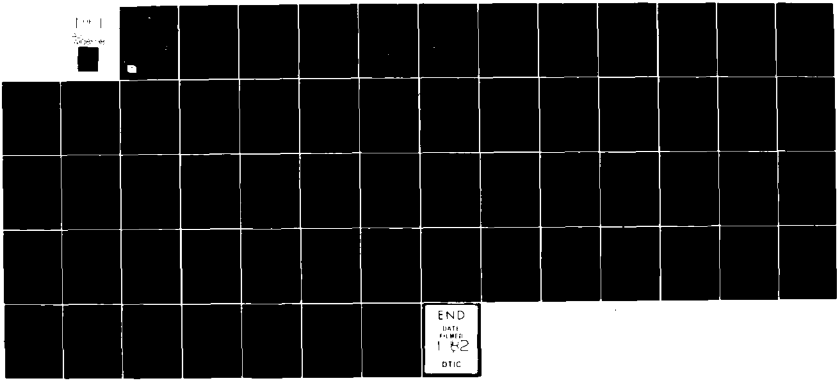

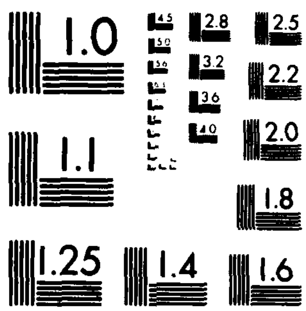

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A SURVEY OF THE LIMITS ON RADAR IMAGING

Irvin W. Kay

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This paper explores the theoretical limits on performance of millimeter-wave radar imaging systems whose purpose is to recognize by shape and, as far as possible, to identify military targets on the ground. It concludes that, with the possible exception of synthetic aperture radar, such systems, using antennas of a practical size, cannot provide adequate resolution for imaging, as defined in the ordinary sense, tank-size targets over ranges of tactical interest. (continued)

20. (continued)

Among the items considered are target characteristics that affect image quality, the image quality needed for target recognition and identification, and the theoretical relations between optical system parameters and image properties such as depth of field and resolution. Also considered are the appropriateness of various definitions of resolution that appear in the literature, distinctions that must be made between extended targets and point targets, and the differences between coherent and incoherent illumination with respect to resolution.

In addition, this paper considers the possibility of improving image quality, particularly resolution, with the aid of superresolution techniques and apodization. It also estimates the possible reduction in the number of elements in a large antenna array by the use of an optimum array thinning distribution of the elements.

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CONTENTS

EXECUTIVE SUMMARY

This paper explores the theoretical limits on performance of radar imaging* systems whose purpose is to recognize by shape and, as far as possible, to identify military targets on the ground. The analysis stresses the 3.2 mm wavelength case, and tanks are assumed to be typical targets of interest.

The major conclusion is that millimeter-wave radars cannot provide imagery* of sufficient quality to identify or even recognize tank-sized targets at tactically interesting ranges without making use of antennas that are too large to be considered feasible. A possible exception is the synthetic aperture radar, which, however, requires an airborne platform, and for which the image quality will be vulnerable and highly sensitive to unpredicted target motion.

In addition to imaging system parameters, the investigation considered target characteristics that affect image quality. It also considered what image quality is needed for target recognition and identification.

When the surface of an extended target is entirely smooth, imaging is not possible. Ideally, for imaging the surface

*In the context of this paper "image" is to be understood in the narrow sense of ordinary usage. That is, the term refers here to the kind of geometrical representation that the human eye ordinarily views and interprets. Intentionally excluded are more general means of identification that are often proposed, such as, for example, the correlation of various radar measurements with known target signatures.

should be uniformly rough relative to the wavelength. That is, it should be covered densely with irregularities, on each of which there is a distinct specular point of reflection. Although the irregularities and the spacing between them must be smaller than a resolution element, at each specular point the principle radii of curvature of the target surface should, nevertheless, be larger than about 1.5 wavelengths.

The number of distinct scattering centers that can exist on a target is limited by the wavelength. When the resolution of an imaging system is sufficiently fine, the image provided by the return from those scattering centers will not appear filled but, rather, will appear to consist of separated points. This suggests that once such a resolution has been reached additional fineness may be of little benefit.

Thus, the value of achieving fine resolution will depend upon the target. For example, aircraft tend to be relatively smooth and therefore may be difficult to recognize even with the fine resolution for which millimeter-wave systems have a potential capability. On the other hand, existing scattering data indicate that ground vehicles such as tanks will provide improved imagery with the resolution improvement that millimeter-wave radar can achieve.

As long as the signal-to-noise ratio is good, the resolution needed for identifying tank-sized targets is such as to provide about 6 line pairs across the smallest target dimension. For the case of primary interest here, this is equivalent to a requirement that the imaging system be able to detect modulation at spatial frequencies at least as high as 3 cycles/m, which is equivalent to a minimum spatial period at least as small as 0.33 m. The requirement for recognition is not significantly lighter, since experiments show that for that purpose about 4 line pairs across the target's smallest dimension are needed.

A review of what is known in general about imaging indicates that predictions sometimes made concerning image quality, based on classical definitions of the optical resolution limit, may be too optimistic. Two reasons for this are evident: failure to take into account the effect of the signal-to-noise ratio and failure to recognize the difference in effect between coherent and incoherent illumination.

In this connection it is important to distinguish between extended targets and point targets. The classical resolution limits refer to point targets. They are valid for extended targets only when the illumination is incoherent and the signal-to-noise ratio is large, i.e., of the order of 20 dB or more. In characterizing the image quality for an extended target, the modulation transfer function is more pertinent than resolution.

This paper also considers optimization methods that might extend the limits on image quality or contribute to the feasibility of an imaging system. Some attention was given to the possibility of improving resolving power by means of superresolution techniques and to the question of how many elements would be required for an antenna array if nonuniform spacing of the elements were permitted.

Investigation disclosed that superresolution techniques will not improve the image quality of an extended target, although apodization (aperture weighting) may reduce the effect of diffraction noise by distributing it more evenly. This is similar in effect to the standard technique of eliminating the speckle phenomenon associated with coherent illumination by dithering the target or performing an equivalent operation on some part of the imaging system.

A calculation demonstrates, on the other hand, that it is feasible to reduce the number of elements required in an array of sensors for a staring system (i.e., one that does not scan

the field of view) by distributing the sensors nonuniformly over the aperture. If the field of view is narrow, e.g., restricted to the angle subtended by a single target, the reduction from the number of sensors that would be required if the distribution were uniform can be greater than 50 percent.

I. INTRODUCTION

A. PRIMARY OBJECTIVE AND RESULTS

This paper is a review of the limitations on performance that are inherent in radar imaging systems, particularly if their purpose is to identify military targets on the ground. The emphasis is on 3.2 mm waves used against tank-sized targets.

The primary conclusion of the review is that the only type of millimeter-wave system with a reasonable antenna size that might accomplish the stated purpose is the synthetic aperture radar. With more conventional antennas the required imaging performance is not feasible at tactically useful ranges.

Even at a range of only 1 km, a 3.2 mm radar would need an effective antenna diameter of nearly 20 m to identify a tank. If such an antenna were implemented as an array of sensors and the illumination were restricted to a field of view covering not much more than a single target, the array would still contain a minimum of nearly 17,000 elements.

B. SCOPE AND ANALYTICAL APPROACH

This paper will examine a number of questions keyed to the problem of identifying a tank from imagery produced by a 3.2 mm radar at a range of 1 km. The reason for working with this very short range is that results based on it may be easily scaled to values corresponding to more realistic situations. In addition, the results will act as a conservative estimate or bound on the feasibility of using millimeter-wave radar imaging systems for the intended applications.

It is assumed throughout that propagation conditions are ideal, i.e., that the relative index of refraction of the atmosphere is unity and remains constant over space and time. Therefore, losses in signal strength due to absorption and Mie scattering still remain to be taken into account. This is also true of phase errors due to variations in the refractive index and phase fluctuations due to atmospheric turbulence.

At the relatively short ranges envisioned here, spatial variations of the index of refraction should not be significant. A small amount of turbulence might even be helpful to a system that relies upon coherent illumination because it would introduce phase averaging, which would tend to reduce the speckle effect normally associated with coherent imagery.

The term "image" used in connection with radar sometimes includes within its scope nothing more than a set of signatures by means of which a target can be classified. In this paper the usage is much narrower, implying that the final output of a system defined as imaging is, in fact, the kind of geometrical representation that the human eye ordinarily views and interprets.

Chapter II considers those characteristics of a target that make it a suitable candidate for surveillance by a radar imaging system, as well as the image quality needed for target identification or recognition. Chapter III discusses the system parameters that affect image quality and estimates the limiting parameter values that are required for target identification performance. Chapter IV contains a brief summary of conclusions resulting from the observations and discussions of Chapters II and III.

Appendix A is a tutorial review of general image-processing theory. It is included as a convenient reference for the basic relations and estimates presented in Chapter III and as background material for the array-thinning analysis presented in Appendix B.

An attempt has been made to consider all avenues for reducing system requirements and for improving the fundamental parameters used to measure image quality. Those avenues include, in the first instance, minimizing the number of elements needed in an array of sensors and, in the second, refining resolution by means of superresolution techniques. No attempt has been made, however, to analyze or discuss processing techniques that aim at enhancing the effectiveness of the image display, i.e., techniques, such as selective area averaging or directional filtering, that may properly be classified under the headings of optical data reduction or image enhancement.

II. TARGET CHARACTERISTICS

A. THE NATURE OF RADAR SCATTERING AND IMAGE FORMATION

If the electromagnetic radiation due to a monochromatic radar source is reflected from a large, smooth, plane mirror, the reflected field will appear, in all respects, to come from the geometrical image of the source relative to the mirror. Therefore, the radar return will contain no information about the shape of the mirror except, perhaps, what can be inferred indirectly from the fact that the image of the source is undistorted.

If the mirror is curved--for example, if it is the surface of a large sphere--the return will also appear to come from an image of the source. However, in this case the image will be distorted and blurred. The effect of the change in shape of the mirror surface is to introduce optical aberrations. Nevertheless, it is still the source that is imaged, not the reflector surface.

On the other hand, if the reflector is not smooth but is covered uniformly with small irregularities, energy will appear to be scattered separately from points evenly distributed over the surface wherever it is illuminated. Macroscopically, the return will appear to come from an extended region that is geometrically similar to the illuminated part of the reflector.

When the radar wavelength is small, geometrical optics provides a valid theoretical model of electromagnetic scattering. This is true even when the reflector surface is rough, as long as the principal radii of curvature of most points on the surface

are larger than about 1.5 wavelengths. At the other extreme, microscopic irregularities, i.e., those smaller than an eighth of a wavelength, have a negligible effect on scattering.

Thus, it is reasonable to expect that roughness may be characterized by the existence of a large number of specular points, relative to an imaging system, due to the irregularities distributed over the reflector surface. In fact, statistical models of quasi-diffuse scattering, in good agreement with experiment, have been obtained by assuming that a rough surface consists of randomly oriented surface elements, each of which in accordance with its own orientation reflects in a specular direction relative to the optical rays incident on it (cf. Ref. 1).

At each of the many specular points on a rough reflector surface, a pencil of rays from the incident field generates a pencil of reflected rays that pass through the entrance pupil of the imaging system. A perfect system will process the resulting phase distribution over its aperture to form an image of the caustic* associated with each reflected pencil of rays.

If rays in the pencil are those associated with a spherical wave, the caustic is a point. In that case, the imaging system will produce the diffraction-limited image of a point. If the caustic is not totally degenerate but consists of lines or surfaces, the image may still appear to correspond to a point (of best focus); however, it will also appear to have aberrations, the nature of which will depend upon the caustic geometry.

*A caustic is the envelope formed by the two-parameter family of rays that are orthogonal to a wavefront surface. The caustic is mathematically equivalent to the locus of the principle centers of curvature of the wavefront surface and is, itself, usually a surface consisting of two branches joined along a singular curve called the edge of regression. When the wavefront has appropriate symmetry the caustic may degenerate into a surface and a line, two lines, or even a single point, which would then be the (perfect) focal point of the rays.

When the F-stop of the imaging system is large, aberrations do not greatly affect resolution, at least for target resolution elements near the center of the field of view. Thus, when the return is backscattered, the fact that images of resolution elements in the target surface are actually images of caustics should not, in itself, cause much deterioration of the resolving power of the system. If the illumination is bistatic, however, some deterioration may occur because of coma and greatly increased astigmatism.

For purposes of good imagery a target surface should be uniformly rough; that is, at any aspect the distribution of specular points should be so dense that there will be no regions more than a resolution element or so wide that are devoid of specular points. There are two reasons for this requirement. One is the desirability of having enough point images to fill in the geometrical shapes that characterize the object. The other is to avoid the highly aspect-dependent, excessively large return that may be produced by a smooth surface patch, causing glint or glare which could disrupt the scene or even strain the system's dynamic range capacity.

Other properties of the target may affect image quality, such as lossy surface materials that reduce reflectivity. Reflectivity may also be highly dependent on polarization, e.g., when multiple reflections take place in the scattering process. The dependence of reflectivity on polarization is most pronounced for bistatic illumination, and depolarization can occur in that case even in the absence of multiple reflection.

The intensity of a wave reflected from a specular point depends not only on the reflectivity due to material properties of the reflector, but also on the local curvature of the surface. This geometric dependence is a consequence of the fact that the

intensity* of a propagating wave varies in accordance with the classical inverse square law of geometrical optics.

The generalized version of the law is that the intensity at any point on a wavefront is inversely proportional to the product of the wavefront's two principal radii of curvature at that point. Each of these radii of curvature is the distance from the wavefront along the ray through the point to the ray's point of tangency on one of the two branches of the associated caustic. The law is also equivalent to the conservation of energy flux in every infinitesimal pencil of rays; i.e., it is assumed that no energy can leak from one infinitesimal pencil of rays to another and that energy is conserved.

In the case in which the incident field is a plane wave propagating in a direction normal to the reflector surface, each principal radius of curvature of the wavefront on the backscattered ray at the point of reflection is one-half of a principal radius of curvature of the reflector surface. This leads to the well-known formula for the radar cross section σ due to specular reflection from a large smooth body (cf. Ref. 2), namely,

$$\sigma = \pi \rho_1 \rho_2 \,, \tag{1}$$

where ρ_1 and ρ_2 are the principal radii of curvature of the body's surface at the specular point. The factor π in Eq. 1 results from the definition of radar cross section, according to which the normalized intensity of the scattered field is multiplied by 4π.

*The term "intensity" as used here has the meaning that is common in radar terminology, where it is synonymous with flux density measured in watts per square meter, rather than the meaning that is common in optics, where it refers to the strength of a source and is measured in watts per steradian.

The calculation is more complicated if the scattering is bistatic or the incident wave is nonplanar. However, in that case it is still true that the intensity of the scattered field is proportional to the product of the principal radii of curvature of the reflector surface at the specular point.

B. REQUIREMENTS FOR TARGET RECOGNITION AND IDENTIFICATION

A pragmatic way to determine what image quality--in particular, what resolution and contrast--is needed for the recognition and identification of a given type of target is to analyze the target's distinctive visual characteristics pertaining to shape. In general, the answer will not only vary with the target but also with its aspect.

In the case of a tank viewed broadside the most distinctive features are probably the typical parallelogram shape of its tread, the trapezoidal shapes above the tread, and the helmet-like turret. It is tempting to include the long cylindrical shape of its one or more guns. However, some evidence exists that gun barrels at most aspects may be absent from millimeter-wave imagery of tanks,* probably because gun barrel surfaces are relatively smooth in this wavelength regime.

One characteristic that may be useful for identification purposes is the number of a tank's bogey wheels. Thus, a standard for the resolution needed to identify tanks might be based

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*See Ref. 3, which contains a number of figures showing 3.2-millimeter-wave as well as optical imagery of the same targets, including some that portray a tank. Whenever the target is a tank, the guns, although present in the optical, are missing from the millimeter-wave imagery. Reference 4** contains a data base of synthetic aperture radar imagery on tanks and trucks. Measurements at 35 GHz were made to characterize reflection centers.

**No classified material from this classified reference has been used in this paper.

on what would be required to count its bogey wheels. That resolution would be sufficient for estimating lengths and widths of other parts of the tank to within a tolerance of the order of a bogey wheel radius.

Another approach to estimating the image quality needed to identify or recognize tanks is to apply known results of psychophysical experiments, such as the number of resolution elements that must be perceived in a target image for recognition or identification. Reference 5 discusses in detail such data and conditions under which it may be used reliably.

Reference 5 lists an empirically derived set of rules, due to J. Johnson (Ref. 6), that prescribe the number of resolution line pairs across the minimum target dimension that are required for a hierarchy of discrimination levels. In increasing order, these levels are designated: *detection*, *orientation*, *recognition*, and *identification*. A cautionary is added, emphasizing that in applying the rules care must be taken not to neglect contrast and its dependence, along with resolution, on the signal-to-noise ratio.

Johnson's rule for target recognition is that about 4 line pairs of resolution must exist within the target's smallest dimension. For identification his rule increases that number to about 6 line pairs. The minimum dimension of a tank is generally its height, which may be on the order of 2 meters. For tank identification this translates into a requirement for a target resolution corresponding to a minimum spatial period* of about 0.5 m to about 0.33 m, which is comparable to the radius of a bogey wheel. Thus, Johnson's rule appears to be consistent with the estimate obtained by means of what has been characterized here as the pragmatic approach.

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*See Section III-B-2-e for a discussion of the relationship between spatial fequency and resolution.

III. IMAGING SYSTEMS

A. TYPES OF SYSTEMS

Basically there are two types of radar systems that image extended targets. One is the line scanning system, which actively scans the target surface with a focused beam, integrating the separate return from each resolution element sequentially along a raster. The other is the staring system, active or passive, which receives and processes simultaneously the return from many resolution elements on the target surface, however the target may be illuminated.

For a line scanning system, resolution in the elevation and azimuth dimensions depends upon the beamwidth of the illuminator and/or receiver in the focal region and, therefore, upon the size of the effective aperture(s) associated with the device. For a staring system, the resolution depends upon the size of the aperture at the entrance pupil associated with the receiving sensor.

Either type of radar system may provide imagery over another dimension by ranging. For this purpose various techniques are available: time gating, pulse compression, and a variety of heterodyning schemes that can be used with CW. In principle, it is possible to estimate the range of a resolution element by means of focusing; however, at telescopic distances the depth of field of an imaging system will usually be too large to make this approach practical.

Either type of system may use an array of sources for active illumination or an array of sensors for reception. At least in the millimeter-wave regime, staring systems may have

fewer problems with arrays than scanning systems. For passive reception the array elements can remain co-phased relative to a single local oscillator,* so that phase shifters would not be necessary.

Scanning systems that scan in two dimensions may suffer from ambiguities which, as observed in Ref. 5, can result from the periodic nature of the raster. Removing such ambiguities, e.g., by means of spatial filtering, could degrade resolution or contrast.

Synthetic aperture radar (SAR) is an imaging system having the ability to produce fine-resolution imagery with a small-aperture antenna.** Most of its applications, such as terrain mapping, have been those involving broad-area, two-dimensional targets.

SAR has a fundamental limitation: it can only provide fine-resolution imagery in the range and azimuth dimensions. For this reason and because of the fact that the real antenna must travel along a path at least as long as the synthetic aperture that it generates during the integration time, it is likely that the platform must be airborne in any application.

The most difficult problem for a SAR is generally the phase error caused by deviations of the platform from its prescribed path. If the target is not stationary, the phase error due to target motion creates an equally difficult problem. Sensitivity to these kinds of error scales with wavelength;

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*See Appendix A for image processing details that justify this statement.

**As it is resolving images in azimuth, it operates basically as a staring system. However, since SAR scans in range, it may alsc be regarded as a linear scanning array.

therefore, a millimeter-wave system should be an order of magnitude more sensitive to errors generated by target and platform motion than an X-band system.

In mitigation, it should be pointed out, however, that *since for the same resolution the required aperture length is shorter for millimeter waves, errors that increase or accumulate in time will have less time to do so.* In fact, the effect of such errors can generally be reduced by increasing the radar platform velocity.

SAR systems ordinarily correct for some of the error created by platform motion deviations, to the extent that it is possible to sense such deviations. In principle, with the aid of range rate measurements similar corrections can be made for errors due to the gross translational motion of a target. However, the system appears to be at the mercy of other kinds of target motion such as pitch and yaw, and must rely upon shortening the integration time, as required, to reduce their effect.

B. RELATIONS BETWEEN IMAGE CHARACTERISTICS AND SYSTEM PARAMETERS

1. Fundamental Parameters of Imaging Systems

All imaging systems are subject to the same basic limitations and tradeoffs among the parameters that define image quality and coverage. The possible values of these parameters depend upon a small number of fundamental system constants. They are: the wavelength λ, the limiting aperture (assumed here to be circular with a diameter D), the signal bandwidth $1/\tau$, and the signal-to-noise ratio S/N.

2. Image Characteristics

a. Depth of Field. If a diffraction-limited imaging system is focused on a point located on the optical axis at a distance z from the entrance pupil plane, there will be an interval, called the depth of field, about the point where all other points are also in focus within some defined resolution criterion.

According to Appendix A, the imaging system will be focused on a target resolution element that is in a plane a distance R from the entrance pupil plane as long as R is in the interval determined by

$$z - \frac{2\lambda z^2}{D^2 + 2\lambda z} \leq R \leq z + \frac{2\lambda z^2}{D^2 - 2\lambda z} \qquad (2)$$

when $z < D^2/2\lambda$, where z is the distance between the entrance pupil and the object plane.* When $z \geq D^2/2\lambda$, the upper limit of the interval is ∞ .

For the aperture diameter D sufficiently large compared with the wavelength λ, the interval defined by Eq. 2 is approximately the same as that defined by

$$z - \frac{2\lambda z^2}{D^2} \leq R \leq z + \frac{2\lambda z^2}{D^2} \quad , \qquad (3a)$$

while for large z it is approximately the interval defined by

$$\frac{D^2}{2\lambda} \leq R < \infty \ . \qquad (3b)$$

Although, in principle, the fact that the depth of field is finite could provide a method of ranging, it is clear from Eq. 3 that for millimeter waves and target ranges of 1 km or more the range resolution would be too poor for the method to have any practical value. Figure 1, which shows the variation of the depth-of-field limits, depicting them as functions of D for z = 1 km and λ = 0.0032 m, indicates this graphically.

*The subscript on the variable z used in Appendix A has been dropped here.

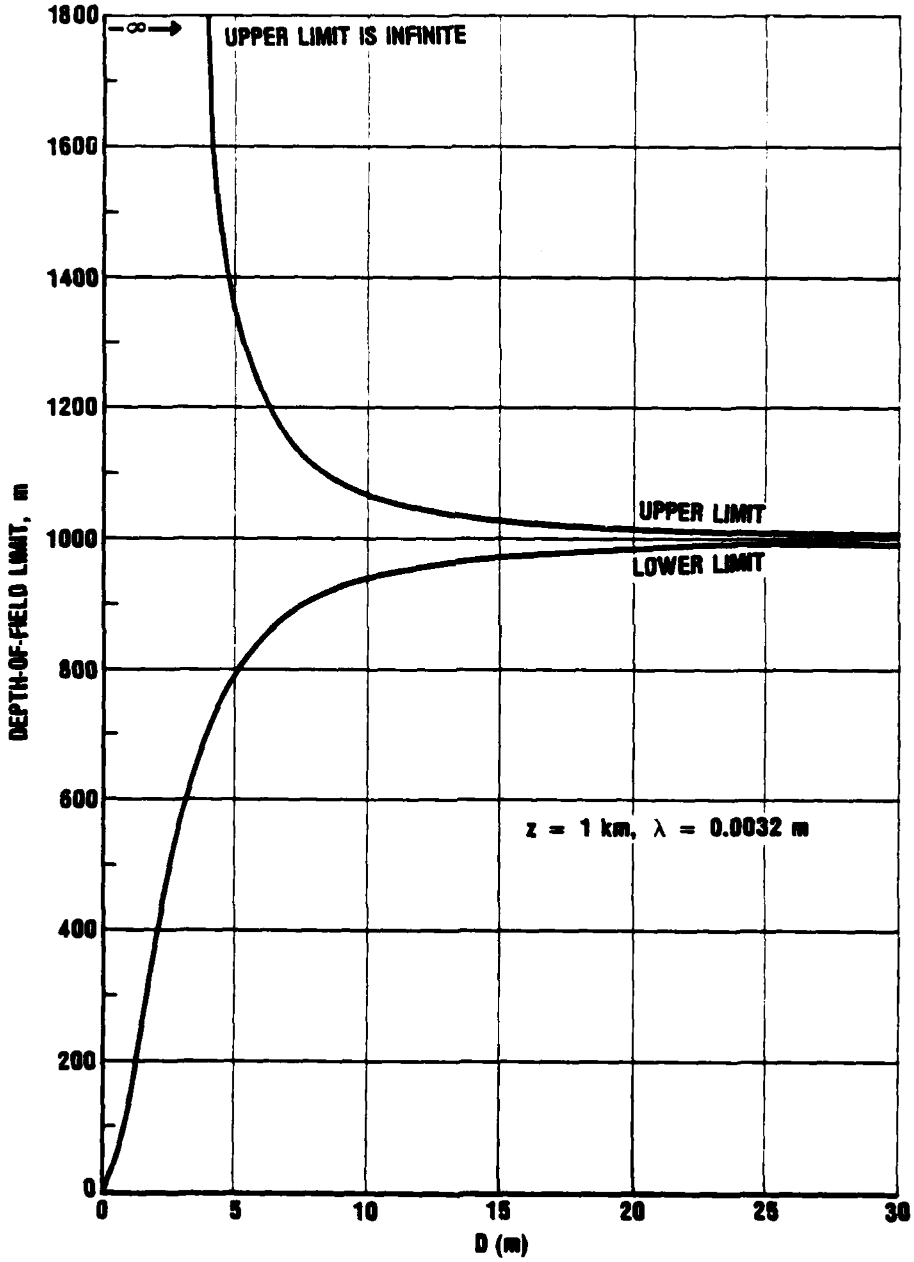

FIGURE 1. Depth-of-field limits.

The derivation in Appendix A of Eq. 2, and therefore Eq. 3, depends upon the Rayleigh criterion for diffraction-limited focusing. Actually, the image intensity drops 19% from its value at the center, i.e., the position of best focus, to its value at either end of the depth of field interval. This, indeed, is one criterion used to define depth of field.

b. Range Resolution. Fine range resolution can be achieved with a short pulse. If the pulse width is τ, the return can be divided into time intervals and processed for range bins, each of which corresponds to a slice of width $\frac{c\tau}{2}$ out of the target, where c is the velocity of light. Thus, a 1-nsec pulse will provide range resolution of 0.15 m.

This implies a signal bandwidth $1/\tau$ of at least 1 GHz. Longer frequency-modulated pulses with the same bandwidth can provide the same resolution when processed by means of pulse compression.

Care must be taken to avoid range ambiguities over the target. If the maximum target dimension in range is ΔR, the pulse repetition rate f_p must satisfy the condition

$$f_p < \frac{c}{2\Delta R} \quad . \tag{4}$$

Thus, for a target extending 10 m in the range direction, ambiguities may occur if the pulse repetition rate exceeds 15 MHz. Of course, if absolute range to the target is desired, ΔR must be replaced by R in Eq. 4.

c. Object Plane Resolution of Point Targets. The resolution limit r_o of two point targets is the minimum separation between their positions such that they can still be recognized as distinct targets. According to the classical Rayleigh definition, if the illumination is incoherent and the object plane is a distance R from the entrance pupil aperture plane,

$$r_o = \frac{1.22\lambda}{D} R \quad . \tag{5}$$

According to Ref. 7,* when the illumination is coherent this resolution limit is degraded by a factor of 1.42; i.e., in that case the resolution limit r_o' is given by

$$r_o' = \frac{1.73\lambda}{D} R \quad . \tag{6}$$

The difference between the resolution limit for the case of coherent illumination and that for the case of incoherent illumination is that the combined pattern for coherent illumination results from the addition of amplitude distributions, whereas the combined pattern for incoherent illumination results from the addition of intensity distributions.

In Ref. 8, R.K. Luneberg defines the resolution limit for two equally bright point targets as the separation at which their image diffraction patterns combine to form a total diffraction pattern which has a single point of maximum intensity, but which for larger separations would have two points of maximum intensity. The resolution limit defined in this way is equal to the distance between the inflection points about the central maximum of the pattern for the image of a single point target.

The resolution limit for the coherent case is equal to the distance between the inflection points about the central maximum of the amplitude pattern for the image of a single point target. For the incoherent case it is equal to the distance between the inflection points about the central maximum of the intensity pattern for the image.

For the case of incoherent illumination, Luneberg's definition leads to the relation

$$r_o = \frac{1.04\lambda}{D} R \quad . \tag{7}$$

*No classified material from this classified reference has been used in this paper.

For the case of coherent illumination, it leads to the relation

$$r'_o = \frac{1.60\lambda}{D} R \quad . \tag{8}$$

Luneberg's definition of the resolution limit implies an error-free measurement of intensity. Thus, the existence of a finite S/N degrades the limit.

Otherwise, S/N determines the maximum number ν of distinguishable levels of intensity, i.e., the dynamic range, in the image. Standard results from information theory provide estimates of ν, which depend upon the noise statistics.

For the case of white noise, for example, it is well known that the information content of a signal amplitude is $\frac{1}{2} \log_2 (1 + S/N)$ bits. Since the sign of the amplitude is lost when only the intensity is known, a measurement of intensity will contain one bit less information. It follows that

$$\nu = \frac{1}{2} \sqrt{1 + S/N} \tag{9}$$

because the information in bits provided by a measurement of intensity is $\log_2 \nu$.

In order to take into account the effect of S/N on resolution, it would seem natural, therefore, to reformulate the definition as follows. The resolution limit for two equally bright point targets is the separation for which the combined diffraction pattern of their images has two points of equal maximum intensity that differ from the intensity at a local minimum between them by an increment that is $1/\nu$ times the intensity at the maxima. This would make the Rayleigh definition for the case of incoherent illumination correspond to an S/N of about 20 dB, since the minimum intensity is about 80% of the maximum in that case.

The resolution limit for SAR (focused) is usually stated as D/2, where D is the diameter of the real antenna. This estimate is actually the distance between half-power points in the effective beam of the synthetic antenna at the range of the resolution element being imaged. However, it agrees very closely with the definition given by Luneberg for extended targets in the coherent case, i.e., Eq. 11 in Section III-B-2-e with D replaced by the synthetic aperture length.

d. Superresolution. A number of authors have proposed schemes for improving resolution by means of phase or amplitude weighting of the return signal distribution over the entrance pupil or some other system aperture (cf. Ref. 9). The idea, generally, has been to reduce the diameter of the central spot in the Airy disc diffraction pattern of a point target image, subject to some optimization condition such as maintaining the greatest possible peak intensity or the maximum energy in the central spot. The peak intensity is always degraded by such a process (Ref. 8), and energy is either absorbed or added to the outer rings of the diffraction pattern.

This type of processing is similar in some respects to monopulse in radar. In fact, one technique, due to J. E. Wilkins (Refs. 9, 10), is the optical analogue of monopulse. Wilkins suggests applying a 180-degree phase shift to the aperture distribution over the central part of the pupil within a circle of specified radius, leaving the rest of the pupil undisturbed. The circular region to which the phase change is applied corresponds to the monopulse sum pattern, and the undisturbed annular ring in the pupil corresponds to the difference pattern. The result is a smaller diameter for the central spot in the Airy disc, with the peak intensity optimized for its greatest possible value.

Reference 9 describes a number of superresolution techniques that affect the point target image diffraction pattern in different ways. Few such techniques have actually been applied, however, although there are some obvious candidates for application.

The improvement of the resolution of double stars has been an objective of image processing by means of aperture weighting in astronomy; however, in practice this seems to have been confined to apodization, a process for which the goal is to reduce the intensity of the outer rings (sidelobes) of the Airy disc. Apodization improves image quality by reducing diffraction noise, but, technically, the effect is to enhance the S/N rather than to improve resolution.

Luneberg (Ref. 8) derived several aperture-weighting functions for superresolution, each optimizing the diffraction pattern with respect to a different criterion. However, he also demonstrated mathematically that, using aperture weighting, it is impossible to increase the image spatial frequency bandwidth, which is limited by the system modulation transfer function (MTF). The implication is that while superresolution is possible, in principle, for point targets, it is not possible for extended targets.

e. Extended Target Resolution. In Ref. 8 Luneberg also proposed an alternative definition of the resolution limit, suitable for an extended target or object plane over which there is a continuous variation of contrast. That definition is directly related to the spatial frequency bandwidth associated with the system MTF.

He showed that the image of a periodic* object plane pattern will be spatially filtered by a system aperture of finite size, so that all Fourier components above a certain frequency in the object plane pattern will be removed. Thus, there will be a minimum spatial period l_o such that the image of a sine wave pattern with a period larger than l_o may exhibit some modulation, but the image of one with a period smaller than l_o will always have zero modulation, i.e., constant intensity across the image plane.

He showed that for incoherent illumination

$$l_o = \frac{\lambda}{D} R \tag{10}$$

and for coherent illumination

$$l_o = \frac{2\lambda}{D} R \quad . \tag{11}$$

Moreover, his demonstration of Eqs. 10 and 11 makes it clear that the result cannot be affected by any kind of aperture weighting. That is, superresolution techniques will not improve resolution for extended objects, although apodization or other kinds of processing may improve contrast or enhance selected spatial frequency components.

This result also implies that the resolution limit for distinguishing details of an extended target is nearly the same as that for distinguishing point targets without the use of superresolution when the illumination is incoherent but is 25% poorer when the illumination is coherent. According to Eq. 11,

*For a finite field of view this is no restriction because the pattern can always be expanded in a Fourier series inside the field of view.

with a 3.2 mm wavelength radar at a distance of 1 km, in order to obtain the minimum spatial period of 0.33 m required for target identification it would be necessary to have an aperture at least 19.4 m in diameter.

f. Field Of View. The field of view may be defined as the angle, measured from the center of the entrance pupil, that is subtended by the part of the extended target for which the system actually produces imagery. Thus, the return from any resolution element on the target within the field of view must contain rays that pass through the entrance pupil aperture. A target point (resolution element) may be outside the field of view either because it is not illuminated or because none of the rays in the pencil of reflected rays that it generates pass through the entrance pupil aperture.*

The pencil of reflected rays generated by each target resolution element in the field of view produces a phase and amplitude distribution over the entrance pupil aperture. Except for aberrations mentioned earlier, the phase contributed to each point on the aperture is proportional to the path length along the ray connecting the resolution element with the point.

The (coherent) electromagnetic field associated with a pencil of rays can be expressed as a superposition of plane waves, each of which, in accordance with its propagation direction, contributes a linear phase variation over the aperture. The range of spatial frequencies, and therefore of propagation directions, of the plane waves that contribute to the imagery finally produced by the system, is limited by the system MTF, which ordinarily vanishes outside a finite pass band. Thus, the MTF limits the system's field of view as well as its resolution and contrast modulation.

*In this connection, it should be observed that the definition of the aperture must take into account any of the system stops that may prevent rays from reaching the image plane.

g. Array Size. As shown in Appendix A, a system can form the image of each target resolution element by applying an appropriate phase correction to the electromagnetic field distributed over the entrance pupil aperture. It is possible to carry out this image processing even if the distribution is only given at discrete sample points as long as the samples are dense enough to satisfy the Nyquist rule: the density must be such that there are at least two samples per cycle at the local spatial frequency corresponding to the aperture phase distribution.

While for a specified resolution the size of the aperture covered by an array of sensors is no different from that required for continuous detection, the number of array elements required and their spacing depends upon the field of view as well as resolution. The total number of elements needed and their density distribution for a given resolution and field of view are calculated in Appendix B. Because the phase distribution contributed by a target resolution element over the entrance pupil is such that spatial frequencies are lower at the center of the aperture than near its perimeter, the element density can be reduced at the center without violating the Nyquist rule. Thus, the total number of elements needed is smaller than would be required to accomodate the maximum spatial frequency in the aperture.

In order to avoid multiple beams, the usual rule for spacing elements in an antenna array is to locate them no more than one-half wavelength apart. According to Appendix B, the total number of elements resulting from an application of this rule will always be greater than the number required for imaging as long as the field of view that must be covered is less than 90 degrees and the distance to the object plane is at least as large as the diameter D of the aperture.

If N_θ is the minimum number of elements required in an array for a field of view equal to θ, and $N_{\frac{1}{2}}$ is the number that

would result from elements spaced one-half wavelength apart, then the ratio of these numbers is given by

$$\frac{N_\theta}{N_{\frac{1}{2}}} = \left(\frac{D}{2z_1}\right)^2 \left(\frac{6n^2 + 8n + 3}{6}\right) , \qquad (12)$$

where z_1 is the distance to the object plane and n is defined by

$$n = \frac{2z_1}{D} \tan\frac{\theta}{2} . \qquad (13)$$

For example, if

$$D = 19.5 \text{ m}, \; z_1 = 1 \text{ km}$$

and

$$\theta = 0.6 \text{ deg},$$

which would suffice to cover a target 10 m long, the ratio would be 1.43×10^{-4}. On the other hand,

$$N_{\frac{1}{2}} = \frac{\pi D^2}{\lambda^2} = 1.17 \times 10^8 \text{ elements}$$

if the wavelength λ is 3.2 mm. Thus, in this case

$$N_\theta = 1.43 \times 10^{-4} \times 1.17 \times 10^8 = 16{,}700 \text{ elements}$$

would be sufficient.

If a uniform distribution of elements over the array is postulated, then their spacing must satisfy the Nyquist rule at the largest spatial frequency in the aperture. In that case, according to Appendix B, the number N_m of elements is related to the ratio $N_\theta/N_{\frac{1}{2}}$ by

$$\frac{N_\theta}{N_m} \sim \frac{N_\theta}{N_{\frac{1}{2}}} + \frac{6n^2 + 8n + 3}{6\ (n + 1)^2} \quad .$$

From this relation it follows that for the example just considered $N_m \sim 26,200$ elements, an increase of 57% over the minimum number that is actually required.

h. SAR Ambiguity Limits. A synthetic aperture radar may produce range ambiguities if the pulse repetition rate f_p is too high and may produce azimuth ambiguities if f_p is too low (cf. Ref. 11). The azimuth ambiguities occur because the sampling rate does not meet the Nyquist rule for the highest-resolution-element Doppler frequencies that are generated during the integration time T.

The effect is to limit the range coverage to a maximum unambiguous range R_u and the platform velocity v, which determines the time that it takes to generate the synthetic antenna length L, i.e., the integration time T. For a system free of ambiguities, these parameters must satisfy the inequalities

$$R_u < \frac{c}{2f_p} < \frac{c\delta}{4v} \quad , \tag{14}$$

where δ is the minimum resolvable distance in azimuth. The second inequality is equivalent to

$$f_p > \frac{2v}{\delta} \quad . \tag{15}$$

If the resolution limit δ required to identify a tank is 0.33 m, then for a platform velocity of 100 m/sec, according to Eq. 15, f_p must be at least 600 Hz, and the unambiguous range R_u obtained from Eq. 14 is 250 m.

In principle, the largest possible platform velocity should be used in order to minimize the integration time, i.e., the time T that it takes to generate the synthetic aperture. Minimizing T should tend to minimize the phase error due to target motion and deviations of the platform from its prescribed path.

The integration time for a target resolution element at range R is given by

$$T = \frac{\lambda R}{vD} \quad . \tag{16}$$

For a target range of 1 km and a real antenna diameter of 25 cm in the example just considered, the integration time is therefore 0.128 sec. Therefore, each target resolution element will be sampled 77 times. Since, for a range of 1 km, increasing f_p by a factor of 250 or less will not cause range ambiguities to develop, it is possible to obtain as much as 23 dB improvement in the S/N by increasing the pulse repetition rate.

IV. CONCLUSIONS

A. CONVENTIONAL IMAGING RADAR CAPABILITY

Because of performance limitations that are inherent in radar imaging systems, as a practical matter the imagery that can be obtained with a wavelength of 3.2 mm will not be good enough for identifying tank-size targets at ranges even as small as 1 km, unless the system used is a synthetic aperture radar. The major impediment is the need for an effective antenna aperture diameter of at least 19.5 m in order to obtain the necessary resolution.

For the most part, tactical vehicle target surfaces should be sufficiently rough (that is, the number of scattering centers should be sufficient) to make detailed imaging possible with millimeter waves at any aspect. For tanks and other gun-bearing vehicles, however, gun barrels are apparently too smooth to appear in the imagery at most aspects, except where the viewing angle is normal to the barrel.

If the radar antenna consists of a two-dimensional array of sensors over a 19.5 m diameter circular aperture, the minimum number of array elements it would require is about 16,700. However, this number will be sufficient only if the illumination can be restricted to a field of view covering not much more than a single target.

The estimate of 16,700 elements for the 19.5 m array diameter is based on the assumption that the spacing between sensors is nonuniform and is no smaller than is required to accommodate the expected variation of phase distribution received over the

aperture. For a uniform spacing sufficient to satisfy the Nyquist rule for the minimum sampling density, the estimate would be 26,200 elements. At least for small fields of view, it is always possible, in principle, to reduce the number of elements required in an array by using nonuniform rather than uniform spacing.

Superresolution techniques exist for improving the resolution of point targets beyond the basic limitation ordinarily imposed by the aperture size. However, such techniques fail when applied to extended targets, for which modulation transfer is a more appropriate measure of image quality than resolution as it is usually defined.

The difference between the limit of resolution in the cases of coherent and incoherent illumination is greater for extended targets than it is for point targets. Thus, the conclusion of V. Corcoran in Ref. 7* regarding this difference (that the resolution limit for the coherent case is 1.41 times as large as for the incoherent case) is not valid for extended targets.

B. SAR CAPABILITY AND POTENTIAL PROBLEMS

In principle, synthetic aperture radar can achieve the image quality necessary for identifying military targets such as tanks at ranges much larger than 1 km. If the targets are stationary, the only essential limitation appears to be the amount of power available.

Phase errors due to unpredicted target and platform motion are always a problem for SAR. Since error sensitivity varies inversely with wavelength, a millimeter-wave system would be an order of magnitude more sensitive to motion effects than would an X-band system. However, for the same resolution the millimeter-wave aperture would be smaller, and therefore errors would accumulate over a shorter time.

*No classified material from this classified reference has been used in this paper.

It should be possible to measure the target's gross translational motion and make appropriate corrections, but effects due to other types of target motion, such as pitch and yaw, will still be a problem. SAR systems ordinarily provide some correction for the various errors due to platform motion.

The effect of target and spurious platform motion on image quality is also a function of the integration time, which should be significantly smaller for the contemplated ranges (1 to 10 km) than it is in the usual X-band applications. Increasing the platform speed will help reduce the integration time still further.

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*No classified material from these references was used in the preparation of this paper.

APPENDIX A

IMAGE PROCESSING FOR A STARING ARRAY

CONTENTS

APPENDIX A

IMAGE PROCESSING FOR A STARING SYSTEM

A. PRELIMINARY CONSIDERATIONS

The scattering of a monochromatic electromagnetic field from a target whose physical dimensions are large compared to the wavelength results in a spatial distribution of the form

$$E_s = \sum_n A_n e^{ik\phi_n} \quad , \tag{A-1}$$

in which each subscript indicates the contribution from a single scattering center on the target. The coefficients A_n in (A-1) are complex vectors that vary slowly over spatial regions, at least where the sensor that detects the field is assumed to be located. The constant k is the wave number, related to the wavelength λ by

$$k = \frac{2\pi}{\lambda} \quad ,$$

and each phase factor ϕ_n is a scalar function that determines a one-parameter family of constant phase surfaces, or wavefronts, given by

$$\phi_n(x,y,z) = \text{constant.}$$

The objective of an imaging system is to separate out the individual terms in (A-1) in order to obtain the contribution of relative intensity, proportional to $|A_n|^2$, and the relative position of each scattering center, which acts as a source for the intensity. The result, ideally, will disclose a uniform

distribution of resolution elements that cover the target image and, by virtue of the intensity variation from one to the other, provide a pattern of contrast that characterizes the target geometrically.

The imaging process relies upon operations performed on the phase functions ϕ_n. For this reason detection of the scattered field must take place over a region that is large enough to accommodate sufficient variation in the ϕ_n to make it possible to distinguish one from another.

It will be assumed that detection of the scattered field takes place within an entrance pupil in the plane z = 0. However, the designation of the entrance pupil is more or less arbitrary; it need not be located where the physical observation of the field actually occurs as long as the amplitude and phase distribution over it is a proper extrapolation of the observed field. Since the sensor area must be of finite extent, the field extrapolated onto the entrance pupil is assumed, in any case, to vanish outside a two-dimensional region referred to as the aperture.

Huygens' principle governs how a field distribution over the entrance pupil would determine the field at a point (x_o, y_o, z_o) on some plane $z = z_o$ within the imaging system. The contribution of a single term in (A-1) would produce a term of the form

$$E_o = \iint_{\text{aperture}} Ae^{ik(\psi+R)}dxdy \quad , \tag{A-2}$$

where

$$R = \sqrt{(x-x_o)^2 + (y-y_o)^2 + z_o^2} \quad .$$

For most positions of the point (x_o, y_o, z_o) the amplitude effect of the variation of the phase $k(\psi+R)$ in (A-2) is assumed to be large compared with the variation in A over the aperture. This phase variation, in fact, causes the integrand to oscillate many times over the integration region so that its contributions to the value of the integral tend to cancel each other. For such positions of (x_o, y_o, z_o) the intensity $|E_o|^2$ given by (A-2) will, therefore, be relatively small.

However, if for other positions of (x_o, y_o, z_o), e.g., in a small region about some point $(\hat{x}, \hat{y}, \hat{z})$, the phase $k(\psi+R)$ is constant or nearly constant over the aperture, all contributions of the integrand to the value of the integral tend to add in phase. In this case the intensity will be relatively large, and the neighborhood of the point $(\hat{x}, \hat{y}, \hat{z})$ may be regarded as a focal region.

The object of image processing is to modify the phase over the entrance pupil in such a way that a separate and distinct focal region is created for each term in (A-1). That is, each ϕ_n is modified for this purpose by the addition of a phase shift, a process which is accomplished in classical optical systems by means of a lens.

B. PLANE WAVE IMAGING (DISCRIMINATING ANGLES OF ARRIVAL)

The wavefront ϕ = constant, radiated from a single scattering center, will be approximately a plane wave, i.e., will have the form

$$\phi(x, y) = px + qy \tag{A-3}$$

over the aperture if the distance to the scattering center is sufficiently large compared to the aperture diameter D.* In (A-3) p and q are direction cosines of the propagation direction

*The distance should be of the order $\frac{2D^2}{\lambda}$ or more, which guarantees that the error in ϕ across the aperture is no greater than $\frac{\lambda}{4}$.

and, as such, provide the angle of arrival of the wave. That is, the wave propagates in the direction of the unit vector with components (p, q, r), where

$$r = \sqrt{1-p^2-q^2} \quad . \tag{A-4}$$

The definition (A-4) implies that

$$p^2 + q^2 \leq 1 \ . \tag{A-5}$$

Plane wave images, after processing, will appear in the system's focal plane located at

$$z_o = f \ ,$$

where f is the focal length. It will be assumed that the focal plane is in the Fresnel diffraction region of the entrance pupil, i.e., f is large enough compared to the diameter D of the aperture so that the approximation

$$R \sim \frac{x^2 + y^2}{2f} - \frac{\left(x_o\, x + y_o\, y\right)}{f} + \frac{x_o{}^2 + y_o{}^2}{2f} + f \tag{A-6}$$

is valid in (A-2). This is equivalent to requiring that the F-stop, which is the ratio of the focal length to the aperture diameter, be large.*

The image processing for plane waves consists of modifying the phase from ϕ, given by (A-3), to

$$\psi = \phi - \frac{x^2 + y^2}{2f} = px + qy - \frac{x^2 + y^2}{2f} \ . \tag{A-7}$$

Then the total phase in the integrand of (A-2) becomes

$$\psi + R \sim \left(p - \frac{x_o}{f}\right)x + \left(q - \frac{y_o}{f}\right)y + \frac{x_o^2 + y_o^2}{2f} + f \tag{A-8}$$

because of (A-6).

*According to the classical Rayleigh criterion, the maximum error in (A-6) across the aperture should be less than $\frac{\lambda}{4}$.

It follows from (A-8) that if x_o, y_o and z_o are coordinates of a point in the focal plane, defined by

$$x_o = fp, \; y_o = fq, \; z_o = f \quad , \tag{A-9}$$

the phase ψ+R is constant over the aperture. Thus, for each p and q that define an angle of arrival there is an image point, given by (A-9), where the intensity appears to be focused.

C. POINT IMAGING

When a target scattering center is a point located at (x_1, y_1, z_1) and is not far enough from the entrance pupil to be in the Fraunhofer region (for which the distance would be of the order $\frac{2D^2}{\lambda}$ or more), then

$$\phi = \sqrt{(x-x_1)^2 + (y-y_1)^2 + z_1{}^2} \quad . \tag{A-10}$$

If the distance is far enough for the target to be in the Fresnel region, however, then an approximation similar to (A-6) is valid, so that

$$\phi \sim \frac{x^2 + y^2}{2z_1} - \frac{x_1 x + y_1 y}{z_1} + \frac{x_1^2 + y_1^2}{2z_1} + z_1 \; . \tag{A-11}$$

In this case the image plane will not be the focal plane and, therefore, f in (A-6) should be replaced by the more general z_o. Nevertheless, the imaging process consists of applying the same phase correction used in (A-7) for the plane waves; i.e.,

$$\psi + R = \phi - \frac{x^2 + y^2}{2f} + R \quad . \tag{A-12}$$

Because of (A-11) and the new approximation for R the total corrected phase is approximately given by

$$\psi + R \sim \alpha \frac{(x^2 + y^2)}{2} + \beta_1 x + \beta_2 y + \gamma \quad , \tag{A-13}$$

where

$$\alpha = \frac{1}{z_1} + \frac{1}{z_o} - \frac{1}{f} \quad , \tag{A-14}$$

$$\beta_1 = -\frac{x_1}{z_1} + \frac{x_o}{z_o} , \quad \beta_2 = -\frac{y_1}{z_1} + \frac{y_o}{z_o} , \tag{A-15}$$

and

$$\gamma = \frac{x_1^{\,2} + y_1^{\,2}}{2z_1} + \frac{x_o^{\,2} + y_o^{\,2}}{2z_o} + z_1 + z_o \quad . \tag{A-16}$$

The coefficient α given by (A-14) will vanish if the image plane is identified with the conjugate plane determined by the lens law of classical optics; i.e., the value of z_o will be determined by

$$\frac{1}{z_1} + \frac{1}{z_o} - \frac{1}{f} = 0 . \tag{A-17}$$

The phase distribution over the entrance pupil will then have the same form, i.e., linear, as in the plane wave case, for which the distribution is given by (A-8). In the present case, however, the coordinates of the image point are determined by setting β_1 and β_2, which are defined by (A-15), equal to zero. The result is

$$x_o = Mx_1 , \quad y_o = My_1 , \tag{A-18}$$

where, because of (A-17),

$$M = \frac{f}{f - z_1} . \tag{A-19}$$

The quantity M is obviously the magnification factor for the image.

D. RESOLUTION

For both the plane wave and the point image cases the field in the appropriate image plane is given by a form of (A-2) in which the total phase of the integrand is a linear function over the aperture because of (A-8) on the one hand and (A-13) on the other. If the slowly varying amplitude factor in (A-2) is

assumed to be constant, then the field in either case will be proportional to a quantity I of the form

$$I = \iint\limits_{\text{aperture}} e^{ik(\xi x+\eta y)}dxdy \ . \tag{A-20}$$

If the aperture is a circular region of radius a, then

$$I = \int_0^a \int_0^{a\pi} e^{ik\rho r\cos(\theta-\theta_o)} r d\theta dr \ , \tag{A-21}$$

where

$$\rho = \sqrt{\xi^2 + \eta^2} \ .$$

The evaluation of the integral in (A-21) is well known (Ref. A-1):

$$I = 2\pi a^2 \frac{J_1(ka\rho)}{ka\rho} \ , \tag{A-22}$$

where J_1 is the Bessel function of order one.

The Rayleigh criterion for the limit of resolution of two point targets is defined as the separation for which the peak of the image plane pattern given by (A-22) for one is at the first null of the image plane pattern for the other. Since the peak value of I occurs where ρ is zero, the smallest solution ρ_o of the equation

$$J_1(ka\rho_o) = 0 \tag{A-23}$$

is the limit of resolution according to the Rayleigh criterion. That solution is the well-known result

$$\rho_o = \frac{0.61\lambda}{a} = \frac{1.22\lambda}{D} \ . \tag{A-24}$$

E. DEPTH OF FIELD

The image of a point target is focused in the conjugate plane determined by the lens law of (A-17). That is, the total phase ψ+R of the integrand in (A-2) is constant over the aperture as long as x_o and y_o satisfy (A-18) and z_o satisfies the lens law (A-17). The image will effectively remain in focus even if z_o deviates from its conjugate plane value by an amount Δz as long as the corresponding deviation in the value of the phase factor ψ+R remains less than the Rayleigh error limit of $\frac{\lambda}{4}$. The interval determined by the excursions Δz that do not violate the Rayleigh limit can be defined as the depth of focus. If the image plane at $z = z_o$ remains fixed and Δz is the excursion of the target from its longitudinal position at z_1 where the object plane is located, the corresponding interval for which the Rayleigh limit is not exceeded can be defined as the depth of field.

Accordingly, if β_1 and β_2 are set equal to zero and z_1 is replaced by $z_1+\Delta z$ in (A-17), then, except for a term that is constant over the aperture,

$$\psi + R \sim \frac{\mu}{2} (x^2 + y^2) , \tag{A-25}$$

where

$$\mu = \frac{\Delta z}{z_1(z_1 + \Delta z)} . \tag{A-26}$$

The maximum phase change occurs at the aperture edge where

$$x^2 + y^2 = a^2 ,$$

and, therefore, is given by

$$\psi + R \sim \frac{\mu}{2} a^2 .$$

At the Rayleigh limit this would be such that

$$\frac{\mu}{2} a^2 = \pm \frac{\lambda}{4} ;$$

i.e.,

$$\mu = \pm \frac{\lambda}{2a^2} = \pm \frac{2\lambda}{D^2} . \qquad (A-27)$$

The depth of field interval is, therefore, defined by

$$z_1 - \frac{2\lambda z_1^2}{D^2 + 2\lambda z_1} \leq R \leq z_1 + \frac{2\lambda z_1^2}{D^2 - 2\lambda z_1} , \qquad (A-28)$$

where R is the distance to the target plane from the entrance pupil plane at $z = 0$ after the target has moved. If

$$D^2 >> 2\lambda z_1 ,$$

then this becomes

$$z_1 - \frac{2\lambda z_1^2}{D^2} \leq R \leq z_1 + \frac{2\lambda z_1^2}{D^2} . \qquad (A-29)$$

The image point field given by (A-2) with (A-25) is proportional to the integral I given by

$$I = \int_0^{2\pi} \int_0^a e^{-ik\frac{\mu}{2}r^2} r dr d\theta , \qquad (A-30)$$

if the aperture is circular and the slowly varying amplitude is assumed to be constant. The integral in (A-30) is easily evaluated:

$$I \sim \frac{2\lambda}{\mu} e^{-i\frac{\pi\mu a^2}{2\lambda}} \sin\frac{\pi\mu a^2}{2\lambda} . \qquad (A-31)$$

At its peak, where $\mu = 0$, the intensity is proportional to

$$|I|^2 = \pi^2 a^4 \quad . \tag{A-32}$$

At the extreme edge of the depth of field interval, where μ has the value given by (A-27), the intensity is proportional to

$$|I|^2 = 8a^4 \quad . \tag{A-33}$$

The ratio of the intensity corresponding to (A-33) to the peak intensity, corresponding to (A-32), is $8/\pi^2$ or 0.81. This corresponds to a drop of 19% for the intensity from its value where the target is at the position of best focus to its value when the target is at the edge of the depth of field interval.

F. REALISTIC SCATTERING CENTER IMAGERY

The image processing discussed in this appendix thus far is based on two assumptions: (1) the scattering center is a point; (2) the F-stop is large enough for the rays that enter the system to be regarded as paraxial. In practice both assumptions will be incorrect to some degree, and to the extent that they are incorrect both lead to phase errors, or optical aberrations, that cause the image plane diffraction patterns to be more complicated than predicted. The effect is to degrade the resolution and make the images appear to be out of focus.

The aberrations due to violation of the second assumption are predictable and well known in classical optics. As past experience has demonstrated, it is possible to correct them when necessary.*

*A general approach that has often been useful in synthetic aperture radar processing is to regard the correction of predictable aberrations as a problem of designing a spatial two-dimensional (or three-dimensional) matched filter.

The aberrations due to the first assumption are most troublesome when the illumination is coherent. They are apparently the cause of the well-known speckle effect, which is averaged out by incoherent illumination. Since they are not predictable, there appears to be no way of eliminating them except by some kind of averaging process, such as dithering the illumination source or possibly the image plane position and orientation.

REFERENCE, APPENDIX A

A-1. M. Kline and I. W. Kay, "Electromagnetic Theory and Geometrical Optics," *Interscience*, New York, London and Sydney, 1965, pp. 434-435.

APPENDIX B

STARING SYSTEM ARRAY THINNING

If the spacing d between the elements of an antenna array is larger than half the wavelength λ, then the detection of the angle of arrival of a plane wave will be ambiguous unless it is known that the field of view is suitably restricted to an angle less than 180 degrees. If it is known that the field of view is no larger than an angle θ, then the detection will be unambiguous as long as

$$d \le \frac{\lambda}{2 \sin\frac{\theta}{2}} . \qquad \text{(B-1)}$$

From another point of view, (B-1) is equivalent to the Nyquist condition for the minimum sampling density required in the array plane in order to reconstruct the incident wave. That is, limiting the field of view to the angle θ is equivalent to limiting the spatial frequency pass band of the signal distribution over the array plane to frequencies bounded by

$$f_{max} = \frac{1}{\lambda} \sin\frac{\theta}{2} . \qquad \text{(B-2)}$$

Therefore (B-1) is just the requirement that the sampling density be at least two samples per cycle at the maximum spatial frequency to be detected by the array elements.

More generally, for any coherent incident wave, if the phase distribution over the array plane is a function $\Phi(x, y)$, then the spatial frequency at a point (x, y) in the direction

of a unit vector $\mathbf{r}_o$ with components $(\cos\phi, \sin\phi)$ is given in terms of the two-dimensional gradient $\nabla\Phi$ by

$$f_s(x, y) = \frac{1}{2\pi} \mathbf{r}_o \cdot \nabla\Phi \ . \tag{B-3}$$

For an incident wave of the form

$$E = A\ e^{ik\psi(x, y, z)}$$

the phase distribution over the array plane is given by

$$\Phi(x, y) = \frac{2\pi}{\lambda} \psi(x, y, 0)\ , \tag{B-4}$$

and therefore, according to (B-3),

$$f_s(x, y) = \frac{1}{\lambda}[\psi_x(x, y, 0)\cos\phi + \psi_y(x, y, 0)\sin\phi]\ . \tag{B-5}$$

For example, in the case of a plane wave

$$\psi(x, y, z) = px + qy + rz\ , \tag{B-6}$$

where p, q, and r are the direction cosines of the wave's propagation direction. Then, according to (B-5),

$$f_s(x, y) = \frac{1}{\lambda}\ (p\cos\phi + q\sin\phi) = \frac{1}{\lambda} \sin\theta_o\ , \tag{B-7}$$

where θ_o is the angle of incidence. Since

$$\theta_o = \frac{\theta}{2}\ ,$$

if the incidence direction is the maximum permitted by the field of view, (B-7) is consistent with (B-2).

If a target point located at (x_1, y_1, z_1) acts as a source for the incident wave, then

$$\psi(x, y, z) = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}\ . \tag{B-8}$$

If $\underline{r}_o$ is chosen to have the direction of $\nabla\Phi$, then f_s will have its maximum value and, according to (B-3) and (B-4), will be given by

$$f_s = \frac{1}{\lambda}|\nabla\psi(x, y, 0)| \ . \qquad \text{(B-9)}$$

From (B-8) and (B-9) it follows that

$$f_s{}^2 = \frac{1}{\lambda^2} \frac{(x-x_1)^2 + (y-y_1)^2}{[(x-x_1)^2 + (y-y_1)^2 + z_1{}^2]} \ , \qquad \text{(B-10)}$$

which can also be written

$$f_s{}^2 = \frac{1}{\lambda^2}\left[1 - \frac{z_1{}^2}{(x-x_1)^2 + (y-y_1)^2 + z_1{}^2}\right] . \qquad \text{(B-11)}$$

Suppose that the array aperture is circular. From (B-11) it is evident that the maximum value of $f_s{}^2$ on any circle of radius ρ about the center of the aperture is given by

$$f^2_{max}(\rho) = \frac{1}{\lambda^2} \frac{(\rho+\rho_1)^2}{[(\rho+\rho_1)^2 + z_1{}^2]} \ , \qquad \text{(B-12)}$$

where

$$\rho_1 = \sqrt{x_1{}^2 + y_1{}^2} \ .$$

This is shown in Fig. B-1.

Now the Nyquist rule that there must be at least two samples per cycle of spatial frequency in any direction will be satisfied if the area density σ of elements in the array satisfies the relation

$$\sigma \geq 4f^2_{max} \qquad \text{(B-13)}$$

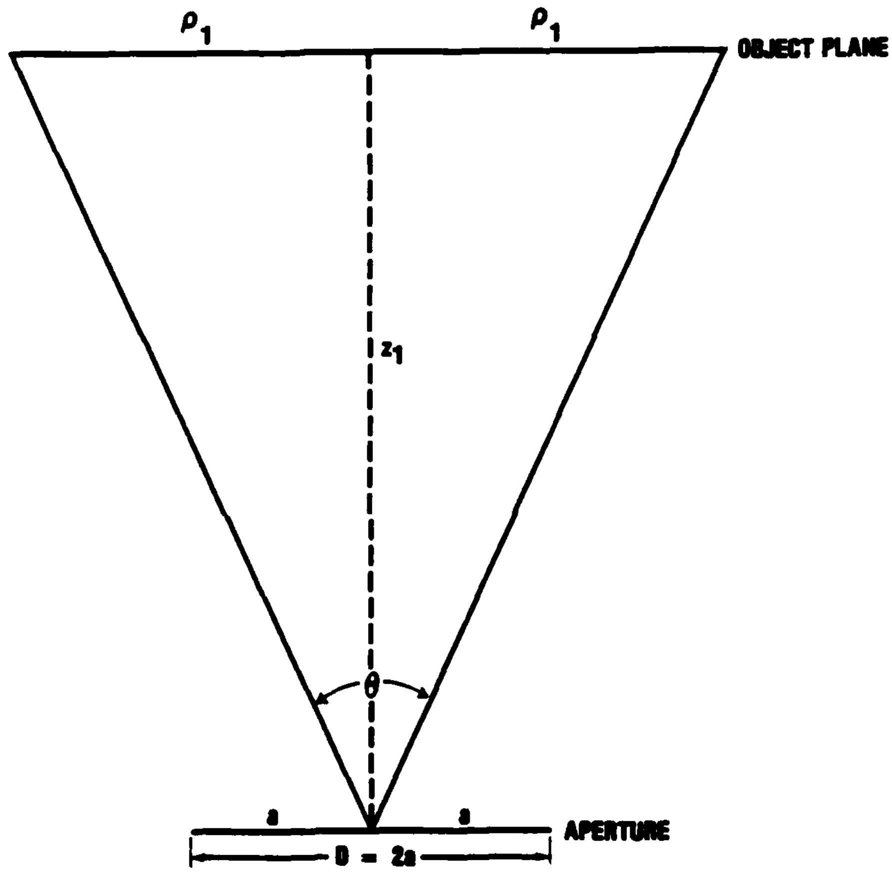

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FIGURE B-1. Field of view for an array of radius a.

on every infinitesimally thin annulus around the center of the aperture. On the other hand, the total number N_e of elements in the array may be defined as the smallest integer such that

$$N_e \geq \iint_{\text{aperture}} \sigma \, dxdy \ . \qquad \text{(B-14)}$$

Then it follows from (B-13) and (B-14) that the Nyquist rule can be satisfied if

$$N_e = 4 \iint_{\text{aperture}} f_{max}^2 \, dxdy + \eta \ , \qquad \text{(B-15)}$$

where

$$0 \leq \eta < 1 \ .$$

For a circular aperture of radius a, after substituting from (B-12) into (B-15), it is found that

$$N_e = \frac{4}{\lambda^2} \int_0^{2\pi} \int_0^a \frac{(\rho+\rho_1)^2}{(\rho+\rho_1)^2 + z_1^2} \rho d\rho d\theta + \eta$$

$$= \frac{8\pi}{\lambda^2} \int_0^a \frac{(\rho+\rho_1)^2}{(\rho+\rho_1)^2 + z_1^2} \rho d\rho + \eta$$

$$= \frac{8\pi}{\lambda^2} \left[\int_0^a \rho d\rho - z_1^2 \int_0^a \frac{\rho d\rho}{(\rho+\rho_1)^2 + z_1^2} \right] + \eta \ .$$

That is, after evaluating the integrals,

$$N_e = \frac{4\pi}{\lambda^2} \left[a^2 + z_1^{\,2}\, U(a,\, z_1) + 2\rho_1 z_1\, V(a,\, z_1)\right] + \eta \,, \qquad \text{(B-16)}$$

where

$$U(a,\, z_1) = \log\left(1 + \frac{\rho_1^{\,2}}{z_1^{\,2}}\right) - \log\left[\frac{(a + \rho_1)^2}{z_1^{\,2}} + 1\right]$$

and

$$V(a,\, z_1) = \tan^{-1}\frac{a + \rho_1}{z_1} - \tan^{-1}\frac{\rho_1}{z_1} \quad .$$

From the fact that for small ε

$$\log\,(1 + \varepsilon) \sim \varepsilon - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

and

$$\tan^{-1}\varepsilon \sim \varepsilon - \frac{\varepsilon^3}{3} + \ldots$$

it follows from (B-16) that for large z_1, i.e.,

$$z_1 >> a + \rho_1 \quad ,$$

$$N_e \sim \frac{2\pi a^4}{(\lambda z_1)^2}\left(\frac{6n^2 + 8n + 3}{3}\right), \qquad \text{(B-17)}$$

where

$$n = \frac{\rho_1}{a} \quad .$$

For elements spaced one-half of a wavelength apart the linear density would be $\frac{2}{\lambda}$ and, therefore, the area density

would be $4/\lambda^2$. In that case the total number $N_{1/2}$ of elements over the aperture would satisfy

$$N_{\frac{1}{2}} \sim \frac{4\pi a^2}{\lambda^2} \;.^*$$ (B-18)

Therefore, combining (B-17) and (B-18),

$$\frac{N_e}{N_{\frac{1}{2}}} \sim \left(\frac{a}{z_1}\right)^2 \left(\frac{6n^2 + 8n + 3}{6}\right) ,$$ (B-19)

where

$$n = \frac{\rho_1}{a} \quad .$$ (B-20)

The ratio is less than one as long as

$$n < \sqrt{\left(\frac{z_1}{a}\right)^2 - \frac{1}{18}} - \frac{2}{3} \sim \frac{z_1}{a} \quad ,$$ (B-21)

i.e., as long as

$$\rho_1 < z_1 \quad ,$$

which is equivalent to the field of view being less than 90 degrees.

Another way to satisfy the Nyquist rule would be to use a uniform density of elements over the aperture with the density equal to what would be required for the largest spatial frequency

*This estimate is conservatively small because the density should be somewhat higher in order to account for the wider spacing that would occur in the diagonal direction.

observed by any part of the array. According to (B-12), the largest spatial frequency is given by

$$f_{max}(a) = \frac{a + \rho_1}{\lambda\sqrt{(a + \rho_1)^2 + z_1^2}} \tag{B-22}$$

$$= \frac{(n + 1)a}{\lambda\sqrt{(n + 1)^2 a^2 + z_1^2}}$$

after a substitution based on (B-20). The corresponding element density is given by $4f_{max}^2(a)$, and therefore the total number N_m of elements in the array is given by

$$N_m \sim 4\pi a^2 f_{max}^2(a) = \frac{4\pi a^4 (n + 1)^2}{\lambda^2[(n + 1)^2 a^2 + z_1^2]} \tag{B-23}$$

because of (B-22). From (B-17), (B-19), and (B-23), it follows that

$$\frac{N_e}{N_m} \sim \frac{N_e}{N_{\frac{1}{2}}} + \frac{6n^2 + 8n + 3}{6(n + 1)^2} . \tag{B-24}$$

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